The Convoy Challenge Problem

Seventh and Eighth Grade Math

Developed by:
The teachers, students, and mentors in the
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1. CONVOY PROBLEM

Primary Resource: Susan Matthews – Wright State University

Adapted by Chris Vanderhorst and Adam Timmerman

1.1. INTRODUCTION

Great Britain has a total of 1,600 nautical miles to guard, measured from headland to headland, 20 miles offshore; 119 ports, large and small, to seal up, 80 of which, even at low tide, are open to vessels that can navigate 14 feet of water; and a larger number of bays and other navigable indentations to watch than are to be found anywhere else in the world in the same length of straightaway shorelines. With all of this taken into consideration it seems impossible for Germany to blockade the British ships.

The United Kingdom's shorelines may be gathered from the map on page 85.

England is so deeply indented that no part is more than 75 miles from the sea, while Scotland has the curviest shoreline of any country in the world.

Ireland is not as deeply indented as England and Scotland; but all its shores make the way of the blockader difficult.

The vast proportions of the British shipping industry, which the German submarine blockade is attempting to destroy, defies comprehension. In normal years, an average of 214 ships arrive at United Kingdom ports from foreign waters every day. In addition, there are 780 arrivals from home ports every day in the year of ships in the coastwise trade.

British merchant ships have a greater total tonnage than those of all the other countries of the world together. The merchant marine of that nation includes nearly 12,000 ships of all kinds. Of these, about 2,800 are sailing ships and 5,300 steam vessels employed in the home trade. There are approximately 4,000 ships engaged in sailing between British and foreign ports. These latter ships have an average capacity of more than 2,500 net register tons.

Great Britain has rapidly been replacing the losses sustained by her shipping as a result of Germany's submarine attacks is disclosed by the fact that at the end of 1916 there were 465 steam vessels under construction in British shipyards, more than half of them being ships of more than 5,000 tons burden. The total capacity of these ships is 1,788,000 tons, so that both in tonnage and in number the new craft are replacing those sunk by the enemy.

The United Kingdom is heavily dependent on its imports, and for her not to have the strongest navy in the world would leave her extremely vulnerable. Approximately, 90 percent of all the food her 45,000,000 people consume is brought in by ships engaged in foreign trade.

On the other hand, Britain has a large supply of coal and iron, which makes British industries largely free from blockade dangers. Producing one-fourth of the world's coal, the United Kingdom has little to fear from a coal shortage, regardless of a blockade.

The port of London handles approximately one-third of all the exports and imports of the United Kingdom.

Liverpool has some of the best docks in the world. Covering the Mersey River for a distance of seven miles, the 60 docks, having 26 miles of quay and covering 428 acres of ground, are equipped with every aid known to industry for the rapid handling of the immense quantities of merchandise.

Cardiff is far down the list in the number of ships arriving, but ranks third in the total tonnage—this is due to the very heavy coal business from that port. Cowes has 24,000 ships a year; Newcastle, 13,000; Portsmouth, 15,000, and Glasgow and Belfast 11,000 each.

As has been said, the British Isles contain no less than 119 ports available for commerce, and practically all of them have been developed for effective use.

Even if the Germans have 500 submarines constructed for the purposes of this blockade, as is claimed, the total makes an average of only about four submarines available for blockading each port.

Submarines, with even the largest radius which any of these boats possess, are dependent upon a convenient base or upon the service rendered by a "mother ship." They generally carry a limited number of torpedoes, without which they are ineffective, and in addition they are severely handicapped by the very nature of their operations.

The ordinary blockade is not subject to these limitations. A blockade established upon the surface of the ocean can maintain a constant lookout over a wide expanse of the sea. By use of searchlights, it can be carried on at night as well as by day. Cruisers may be coaled at sea and provided with ammunition openly. The submarine may not. Without a base or a hovering fleet of "motherships," the submarine cannot do continuous duty on blockade or otherwise.

If it is planned to operate the submarine blockade of the British Isles in relays, the number of ships on duty at a given port will be thereby halved, to the detriment of the blockade's effectiveness. Two submarines to a port could hardly maintain a blockade in the condition which the ordinary interpretation of international law has required to give it recognition among neutrals.

British domination of the sea has not come about by chance. England's geographic limitations have compelled her to keep the avenues of ocean traffic open through constant readiness to render naval protection to her carrying trade; and it is the result of her insular position that her activities have developed on sea and land.

To this they owe the great number of ports which they now possess and which, by their very numbers, render a blockade, however attempted, a herculean task. A clearer example of how nations are limited or advanced by their geographic environment could hardly be found.

1.1.1. ADDITIONAL RESOURCES

This is a really good movie on U-Boats attacking merchant ships!
http://www.youtube.com/watch?v=IIMcT9OOxDY

Below is a link to an article arguing against convoy
Prelude to a Battle – PDF Form

1.2. PROBLEM

For this problem, you'll want to answer the question: : Should britain convoy their merchant ships during WWI? After considering the above reading with its arguments against convoying, you should find that modeling the situation mathematically is helpful in determining the solution to the problem of convoying. Your project must be written by the entire group.

1.3. PARAMETERS

- Because convoys do, indeed, sail at the speed of their slowest ship and take longer to assemble than ships sailing independently, ships sailing alone complete round trips more quickly than convoys. Ships sailing independently take 75% of the time to complete one round trip as compared to the time ships sailing in a convoy take to complete one round trip. Each completed independent round trip took about 4.5 weeks compared to 6 weeks for a convoy. (You may estimate this length of time by using the approximate speed of convoys and of individual ships and the distance from Great Britain to the US.)
- Independently sailing ships lose 14% of their number to submarines on each round trip. In this context, convoys lose 5% of their number on each round trip.
- One round trip means ships either alone or sailing as part of a convoy – Britain to the US or Canada, load supplies and war materiel, return to Britain, and offload. Remember that we shall assume ships sailing independently leave at the same time but take different routes and of course, ships sailing in a convoy leave at the same time.
- Given these parameters, model the situation to determine if merchant ships sailing independently or sailing in convoys can bring the needed relief to Great Britain. Does your answer change over time?

1.4. PROJECT WRITE-UP

- Write your recommendations to the British admiralty for either convoying their merchant ships or continuing to let them sail independently. Include your discussion of the validity of the arguments against convoy, your mathematical models with clear explanations of your work, and information gained from any reading (including outside reading) to support your

recommendations. State any assumptions and simplifications made in the models that you have created and discuss how those might affect your results.

- Think carefully about the arguments against convoy given in the reading. Which seem valid to you and which partially valid? Support your reasons. You may use additional sources to obtain information, but you must cite these sources. Are there any other arguments for or against which ought to have been considered?
- Now include ship replacement in your model. The British could build ships fast enough to replace 5% of the **original number** for each convoy round trip. Note that this means that ships can also be replaced when a ship is sailing independently.
- A major concern in convoying was that the German submarines would be able to find the convoys so much more easily than they could find the individual ships. Is the probability that the submarine detects 1 ship crossing the ocean proportional to how far the submarine can see (i.e. the radius of visibility) or is it proportional to r^2 i.e. to the area of the circle of visibility?
- Also address the following:
  - A way to model this is with a circle around the point representing where the submarine stays on the surface and with a line representing the track of the ship across the ocean.
  - Now consider 10 ships. If they steam independently, there are 10 separate tracks. What is the probability that the submarine detects at least 1 ship of the 10 ships compared to when there was only 1 ship?
  - If the 10 ships steam together in a convoy, the convoy fits in a group close together, and the submarine detects the convoy if it sees any of the ships. What is the probability that it detects the convoy compared to detecting 1 single ship? Explain how you modeled this and how that gives you the answer.
  - What argument does this address?

Section	Points	Notes	Comments:
Statement of the problem	/5	Clear statement of the problem and sufficient background to put it into the appropriate context.	
Arguments about validity of the preliminary arguments against convoying.	/15	Clear response to each argument against convoying. Do not simply paraphrase arguments in the article.	
Assumptions	/10	Clear identification of simplifying assumptions with discussion of how much you think they matter.	
First and Second Models	/30	All calculations and formulas shown; tables clearly labeled, correct, and explained. Strong explanation as requested.	
Probability model	/15	Appropriate and correct calculations with accompanying explanations. Correct application.	

Summary and Conclusions	/15	Clear and based upon your work.	
Presentation	/10	Units when appropriate, sufficiently proofread, correct grammar and spelling, neat. Points will be deducted when needed.	
Total points	100		